Solution
MOCK EXAM - PAPER 3
Class 10 - Mathematics

Section A

1.
(b) 51
Explanation:
867 = 255 $\times$ 3 + 102
255 = 102 $\times$ 2 + 51
102 = 51 $\times$ 2 + 0
Hence, we got remainder as 0 , therefore HCF of (867 , 255) is 51

2.
(b) 3
Explanation:
The number of zeroes is 3 as the graph intersects the x-axis at three points.

3.
(c) consistent with unique solution.
Explanation:
Since the lines in the graph are not parallel, they will be consistent, also they are not coinciding, that means they have unique solution.

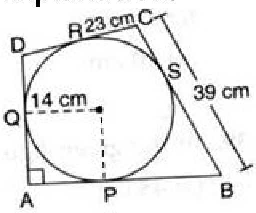
4.
(d) 12$\sqrt{2}$
Explanation:
Equation is $2x^2 + ax + 32 = 0$
Let one root beα, then other would be 2α
Now$\alpha \times 2\alpha$= 16$\Rightarrow \alpha = \pm 2\sqrt{2}$
andα+ 2α= $\frac{-a}{2} \Rightarrow 3\alpha = \frac{a}{2} \Rightarrow 6\alpha$= -a
$\Rightarrow \pm 12\sqrt{2}$= -a or a =$\pm 12\sqrt{2}$

5. **(a)** 16^{th}
Explanation:
a = -29
d = 3
a_n = 16
16 = a + (n - 1)d
16 = -29 + (n - 1)3
$\frac{45}{3} = n - 1$
15 + 1 = n
n = 16
a_{16} = 16

6. **(a)** $\sqrt{41}$
Explanation:
The distance of the point (5, 4) from origin
$= \sqrt{5^2 + 4^2}$
$= \sqrt{25 + 16} = \sqrt{41}$ unit

7. **(a)** 0
Explanation:
Let the points A (2, 3), B(4, k) and C(6, -3) be collinear.
If the points are collinear then the area of triangle ABC formed by these three points is 0.
$\therefore$ ar($\triangle$ABC) = $\frac{1}{2}$ |$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$| = 0
$\Rightarrow$ 2 = $\frac{1}{2}$ |(k + 3) +4(-3 - 3) + 6(3 - k)| = 0
$\Rightarrow$ |2k + 6 - 24 + 18 - 6k| = 0
$\Rightarrow$ |-4k| = 0
$\Rightarrow$ k = 0

8. **(c)** $\frac{9}{2}$
Explanation:
According to Basic Proportionality theorem.
$\frac{AD}{DB} = \frac{AE}{EC}$
$\frac{2}{3} = \frac{3}{x}$
x = $\frac{9}{2}$

9. **(d)** 30 cm
Explanation:


Let the centre of the circle be O.
Construction: Joined OP.
$\because$ Tangent is perpendicular to the radius through the point of contact.
$\therefore \angle$OQA = $\angle$OPA = 90° and OQ = OP [Radii]
$\therefore$ OQAP is a square.
$\therefore$ AP = 14 cm
Now, CR = CS = 23 cm [Tangents from an external point to a circle are equal]
$\therefore$ BS = 39 - 23 = 16 cm
And BS = BP = 16 cm [Tangents from an external point to a circle are equal]
Now, AB = AP + BP = 14 + 16 = 30 cm

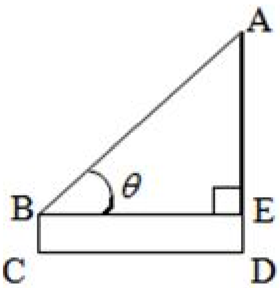
10. **(a)** two points
Explanation:
two points

11. **(d)** cosec A + cot A
Explanation:
$$\sqrt{\frac{1+\cos A}{1-\cos A}} = \sqrt{\frac{(1+\cos A)}{(1-\cos A)} \times \frac{(1+\cos A)}{(1+\cos A)}} = \frac{(1+\cos A)}{\sqrt{1-\cos^2 A}} = \frac{(1+\cos A)}{\sqrt{\sin^2 A}}$$
$$= \frac{(1+\cos A)}{\sin A} = \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A}\right) = (cosec\ A + \cot A)$$

12. **(d)** 3

Explanation:
Since, we know
$Sec^2 \theta$ - $tan^2 \theta$ = 1
(Secθ + tanθ) (Secθ - tanθ) = 1
(Secθ + tanθ)$\frac{1}{3}$ = 1
Secθ + tanθ = 3

13.
(b) 45°
Explanation:



Let θ be the angle of elevation,
The height of the tower AD = 25 m
And CD = 23.5 m
In triangle ABE,
$\therefore \tan\theta = \frac{\text{AE}}{\text{BE}} = \frac{\text{AD}-\text{ED}}{\text{CD}}$
$\Rightarrow \tan\theta = \frac{25-1.5}{23.5} = \frac{23.5}{23.5} = 1$
$\Rightarrow \tan\theta = \tan 45^\circ \ \theta$
$\Rightarrow \theta = 45^\circ$

14.
(c) 72^o
Explanation:
It is given that area of the sector = 69.3 cm^2
and Radius = 10.5 cm
Now, Area of the sector $= \frac{\pi r^2\theta}{360}$
$\Rightarrow \frac{\pi\times(10.5)^2\times\theta}{360}$ = 69.3
$\Rightarrow \theta = \frac{69.3\times360\times7}{10.5\times10.5\times22}$ = 72^o
Therefore, Central angle of the sector = 72^o

15.
(b) $10\sqrt{2}$ units
Explanation:
$10\sqrt{2}$ units

16.
(d) - 1.5
Explanation:
– 1.5 cannot be the probability of an event because $0 \leqslant P(E) \leqslant 1$.
The probability of a sure event is 1 and the probability of an impossible event is 0.

17.
(c) $\frac{11}{36}$
Explanation:
Elementary events are
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)
$\therefore$ Number of Total outcomes = 36
And Number of possible outcomes = 11
$\therefore$ Required Probability = $\frac{11}{36}$

18. **(a)** 17.5
Explanation:
Given, classes are not continuous, so we make continuous by subtracting 0.5 from lower limit and adding 0.5 to upper limit of each class.

Class	Frequency	Cumulative frequency
-0.5-5.5	13	13
5.5-11.5	10	23
11.5-17.5	15	38
17.5-23.5	8	46
23.5-29.5	11	57

Here, $\frac{N}{2}=\frac{57}{2}=28.5$, which lies in the interval 11.5 - 17.5.
Hence, the upper limit is 17.5.

19.
(d) A is false but R is true.
Explanation:
A is false but R is true.

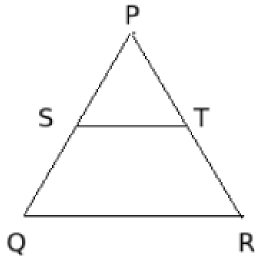
20.
(d) A is false but R is true.
Explanation:
A is false but R is true.

Section B

21. HCF of two or more positive integers is the largest positive integer that divides all the numbers exactly.
We can find the H.C.F by applying division lemma on 18 and 24
Clearly, 24 = 18 $\times$ 1 + 6.
Since remainder is not zero, applying division lemma on divisor 18 and remainder 6 gives
18 = 6 $\times$ 3 + 0.
Therefore, H.C.F. of 18 and 24 is 6.

OR

Let us assume that $5+2\sqrt{7}$ is not an irrational number.
$\therefore 5+2\sqrt{7}$ is a rational number p i.e. $5+2\sqrt{7}$ = p
$\Rightarrow \sqrt{7}=\frac{p-5}{2}$
This is a contradiction as RHS is rational but LHS is irrational.
Hence $5+2\sqrt{7}$ can not be rational, so irrational.

22. 

Given: In ΔPQR, PS = 2.4 cm, SQ = 7.2 cm, PT = 1.8 cm and TR = 5.4 cm,

We know that if a line divides any two sides of the triangle in the same ratio, then the line is parallel to the third side.

i.e. to prove ST||QR we have to show that,

$\frac{PS}{SQ} = \frac{PT}{TR}$

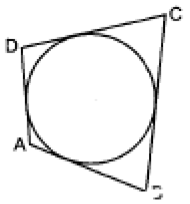
LHS = $\frac{PS}{SQ}$ = $\frac{2.4}{7.2}$ = $\frac{1}{3}$

RHS = $\frac{PT}{TR}$ = $\frac{1.8}{5.4}$ = $\frac{1}{3}$

Since LHS = RHS

Hence, ST||QR

23. Given, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm.



If a circle touches all the four sides of quadrilateral ABCD, then

AB + CD = AD + BC

$\therefore$ 6 + 8 = AD + 9

$\Rightarrow$ 14 = AD + 9

$\Rightarrow$ 14-9 = AD

$\Rightarrow$ AD = 5 cm

24. Dividing N^r & D^r by sin A in LHS

= $\frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$

= $\frac{\cot A + \operatorname{cosec} A - \left(\operatorname{cosec}^2 A - \cot^2 A\right)}{\cot A + 1 - \operatorname{cosec} A}$

= cosec A + cot A

OR

We have,.

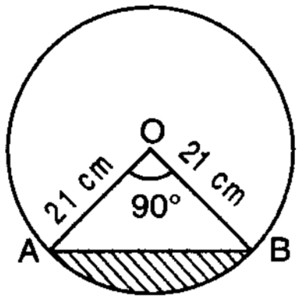
tan θ + cot θ = 2

$\Rightarrow$ (tan θ + cot θ)^2 = 4 [On squaring both sides]

$\Rightarrow$ $\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 4$

$\Rightarrow$ $\tan^2\theta + \cot^2\theta + 2 = 4$ [$\because \tan\theta\cot\theta = 1$]

$\Rightarrow$ $\tan^2\theta + \cot^2\theta = 2$

25. 

Area of the sector AOB = $\frac{\theta\pi r^2}{360^\circ}$

$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$

$= \frac{11 \times 3 \times 21}{2} \mathrm{cm}^2$

= 346.5 cm^2

Area of the $\triangle$AOB = $\frac{1}{2} \times \mathrm{OA} \times \mathrm{OB}$

$= \frac{1}{2} \times 21 \times 21 \mathrm{cm}^2$

= 220.5 cm^2

Area of the segment = area of the sector AOB - area of the $\triangle$AOB

= 346.5 - 220.5 cm^2

= 126 cm^2

Section C

26. This problem can be solved using Least Common Multiple because we are trying to figure out when the soonest (Least) time will be that as the event of exercising continues (Multiple), it will occur at the same time (Common).

L.C.M. of 12 and 8 is 24.

So,

They will exercise together again in 24 days.

27. $7x^2 + 18x - 9$

$= (7x - 3)(x + 3)$

$\therefore$ Zeroes are $-3, \frac{3}{7}$

New zeroes are $-6, \frac{6}{7}$

Sum of new zeroes $= (-6) + \frac{6}{7} = -\frac{36}{7}$

Product of new zeroes $= (-6) \times \frac{6}{7} = -\frac{36}{7}$

$\therefore$ Required polynomial is $x^2 + \frac{36}{7}x - \frac{36}{7}$ or $7x^2 + 36x - 36$

28.

Percentage of marks (Class Interval)	**Number of students (f_i)**	**Cummulative frequency**
30-35	16	16
35-40	14	30
40-45	18	48
45-50	20	68
50-55	18	86
55-60	12	98
60-65	2	100

Now, N= 100 So, $\frac{N}{2}$ = 50

$\therefore$ Median class is 45-50

So, l = 45,

cf (cummulative frequency of preceding class) = 48

f (frequency of median class) = 20

h = 5

∴ Median = $l+\left(\frac{\frac{n}{2}-cf}{f}\right)\times h$

Median = $45+\left(\frac{50-48}{20}\right)\times 5$

Median = $45+\left(\frac{2}{4}\right)$

Median = 45 + 0.5

Median = 45.5

29. Given, 99x + 101y = 499 ....(i)

101x + 99y = 501... (ii)

Adding eqn. (i) and (ii),

(99x + 101y) + (101x + 99y) = 499 + 501

99x + 101y + 101x + 99y = 1000

200x + 200y = 1000

x + y = 5 ...(iii)

Subtracting eqn. (ii) from eqn. (i), we get

(99x + 101y) - (101x + 99y) = 499 - 501

99x + 101y - 101x - 99y = -2

-2x + 2y = - 2

or, x - y= 1 .... (iv)

Adding equations (iii) and (iv)

x + y + x - y = 5 + 1

2x = 6

∴ x = 3

Substituting the value of x in eqn. (iii), we get

3 + y = 5

y = 2

Hence the value of x and y of given equation are 3 and 2 respectively.

OR

Let the present ages of Baljeet and Amit be x years and y years respectively.

Then,

Baljeet's age 5 years ago = (x - 5) years

and Amit's age 5 years ago = (y - 5) years

∴ (y - 5) = 3(x - 5) $\Rightarrow$ 3x - y = 10 ....(i)

Baljeet's age 10 years hence = (x +10) years

Amit's age 10 years hence = (y +10) years

∴ (y + 10) = 2(x + 10) $\Rightarrow$ 2x - y = - 10 ....(ii)

On subtracting (ii) from (i), we get x = 20.

Putting x = 20 in (i), we get

(3 $\times$ 20) - y = 10 $\Rightarrow$ y = 60 - 10 = 50.

∴ x = 20 and y = 50.

Hence, Baljeet's present age = 20 years

and Amit's present age = 50 years.

30. LHS: $\frac{\cos^2\theta+(1+\sin\theta)^2}{(1+\sin\theta)\cos\theta}$

$=\frac{\cos^2\theta+1+\sin^2\theta+2\sin\theta}{(1+\sin\theta)\cos\theta}$

$=\frac{2(1+\sin\theta)}{(1+\sin\theta)\cos\theta}$

$=2\sec\theta=$ RHS

31. $\angle QPT=75^\circ$

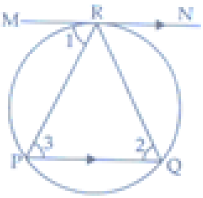
$\angle PQT=75^\circ$

$\theta=30^\circ$

$\sin 2\theta = \sin 2(30°)$
$= \sin 60° = \frac{\sqrt{3}}{2}$

OR

Given: In a circle a chord PQ and a tangent MRN at R such that QP || MRN



To prove: R bisects the arc PRQ.
Construction: Join RP and RQ.
Proof: Chord RP subtends ∠1 with tangent MN and ∠2 in alternates segment of circle so ∠1 = ∠2.
MRN || PQ
∴ ∠1 = ∠3 [Alternate interior angles]
⇒ ∠2 = ∠3
⇒ PR = RQ [Sides opp. to equal ∠s in Δ RPQ]
∵ Equal chords subtend equal arcs in a circle so
arc PR = arc RQ
or R bisect the arc PRQ. Hence, proved.

Section D

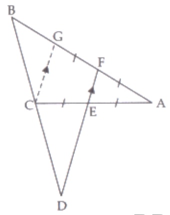
32.

Number of lions	Number of regions	Cumulative frequency
0-100	2	2
100-200	5	7
200-300	9	16
300-400	12	28
400-500	x	$28 + x$
500-600	20	$48 + x$
600-700	15	$63 + x$
700-800	9	$72 + x$
800-900	y	$72 + x + y$
900-1000	2	$74 + x + y$
	100	

$74 + x + y = 100$
$x + y = 26$
Median class is $500 - 600$

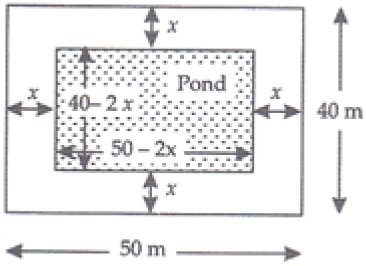
$$525 = 500 + \left[\frac{\frac{50}{2} - (28+x)}{20}\right] \times 100$$

On solving, we get $x = 17$
$y = 9$

33. 

To Prove: $\frac{BD}{CD} = \frac{BF}{CE}$
Construction: Draw CG $\|$ EF.
Proof: In $\triangle AGC\ CG \| EF$

$\because$ E is the mid point of AC
$\therefore$ F will be the mid point of AG.
$\Rightarrow FG = FA$
But, EC = EA = AF [Given]
$\therefore FG = FA = EA = EC$ ....(i)
In $\triangle$BCG and $\triangle$BDF
$EF \| CG$. (By construction)
$\therefore \frac{BC}{CD} = \frac{BG}{GF}$ [By BPT]
$\Rightarrow \frac{BC}{CD} + 1 = \frac{BG}{GF} + 1$
$\Rightarrow \frac{BC+CD}{CD} = \frac{BG+GF}{GF}$
$\Rightarrow \frac{BD}{CD} = \frac{BF}{GF}$
But, FG = CE [From (i)]
$\Rightarrow \frac{BD}{CD} = \frac{BF}{CE}$
Hence, proved.

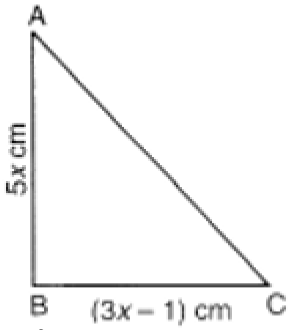
34. 

Let width of the pond be x m. Then,
The length of pond = $(50 - 2x)m$ and the breadth of pond = $(40 - 2x)m$
Area of grass around the pond = 1184 m^2
$\Rightarrow$ Area of Lawn - Area of Pond = 1184
$\Rightarrow 50 \times 40 - (50 - 2x)(40 - 2x) = 1184$
$\Rightarrow 2000 - (2000 - 100x - 80x + 4x^2) - 1184 = 0$
$\Rightarrow 2000 - (2000 - 180x + 4x^2) - 1184 = 0$
$\Rightarrow$ 2000 - 2000 + 180x - $4x^2$ - 1184 = 0
$\Rightarrow$ $4x^2$ - 180x + 1184 = 0
$\Rightarrow 4(x^2 - 45x + 296) = 0$
$\Rightarrow$ x^2 - 45x + 296 = 0
Factorise now,
$\Rightarrow$ x^2 - 37x - 8x + 296 = 0
$\Rightarrow$ x(x - 37) - 8(x - 37) = 0
$\Rightarrow$ (x - 37) (x - 8) = 0
$\Rightarrow$ x - 37 = 0 or x - 8 = 0
$\Rightarrow$ x = 37 or x = 8
When x = 37, then
The length of pond = 50 - 2 $\times$ 37
= 50 - 74
= -24 m (Length cannot be negative)
When x = 8, then
The length of pond = 50 – 2x
= 50 - 2 $\times$ 8
= 50 - 16
= 34 m
And the breadth of the pond
= 40 - 2x
= 40 - 2 $\times$ 8
= 40 - 16

= 24 m
Therefore, the length and breadth of the pond are 34 m and 24 m respectively.

OR

Area of triangle = $\frac{1}{2} \times$ base $\times$ h$eight$
$= \frac{1}{2} \times 5x \times (3x - 1)$
According to the question,

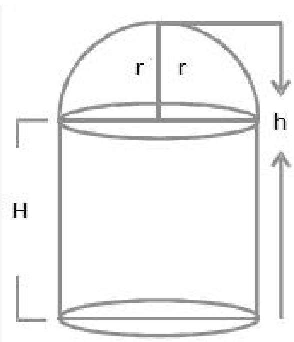


$15x^2 - 5x = 120$
or, $3x^2 - x - 24 = 0$
or, $3x^2 - 9x + 8x - 24 = 0$
or, 3x (x - 3) + 8 (x - 3) = 0
or, (x - 3) (3x + 8) = 0
$\therefore \quad x = 3, x = -\frac{8}{3}$
Length can't be negative, so x = 3
AB = 5 $\times$ 3 = 15 cm, BC = 3x - 1 = 9 - 1
= 8 cm
$AC = \sqrt{15^2 + 8^2}$
$= \sqrt{225 + 64}$
$= \sqrt{289} = 17\text{cm}$
Hence hypotenuse = 17 cm

35. Radius of sphere $= R = 7\ cm$

Radius of cylinder $= r = 1\ cm$
Capacity of the entire glass vessel $= \pi r^2 h + \frac{4}{3}\pi R^3$
$= \frac{22}{7} \times 1 \times 1 \times 7 + \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$
$= \frac{4378}{3} cu.cm$ or $1459.33 cu.cm$

OR

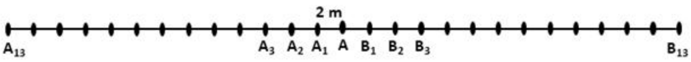


Let the radius of the hemispherical dome be r and the total height of the building be h.
Since, the base diameter of the dome is equal to $\frac{2}{3}$ of the total height
$2r = \frac{2}{3}h$
$\Rightarrow r = \frac{h}{3}$
Let H be the height of the cylindrical position.
$\Rightarrow H = h - r = h - \frac{h}{3} = \frac{2h}{3}$
Volume of air inside the building = Volume of air inside the dome + Volume of air inside the cylinder

$\Rightarrow 67\frac{1}{21} = \frac{2}{3}\pi r^3 + \pi r^2 H$
$\Rightarrow \frac{1408}{21} = \pi r^2 \left(\frac{2}{3}r + H\right)$
$\Rightarrow \frac{1408}{21} = \frac{22}{7} \times \left(\frac{h}{3}\right)^2 \left(\frac{2}{3} \times \frac{h}{3} + \frac{2h}{3}\right)$
$\Rightarrow \frac{1408 \times 7}{22 \times 21} = \frac{h^2}{9} \times \left(\frac{2h}{9} + \frac{2h}{3}\right)$
$\Rightarrow \frac{64}{3} = \frac{h^2}{9} \times \left(\frac{8h}{9}\right)$
$\Rightarrow \frac{64 \times 9 \times 9}{3 \times 8} = h^3$
$\Rightarrow h^3 = 8 \times 27$
$\Rightarrow h = 6$
Thus, the height of the building is 6 m.

Section E

36. i. Distance covered in placing 6 flags on either side of center point is 84 + 84 = 168 m
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$\Rightarrow S_6 = \frac{6}{2}[2 \times 4 + (6 - 1) \times 4]$
$\Rightarrow S_6 = 3[8 + 20]$
$\Rightarrow S_6 = 84$

ii. 

Let A be the position of the middle-most flag.
Now, there are 13 flags (A_1, A_2 ... A_{12}) to the left of A and 13 flags (B_1, B_2, B_3 ... B_{13}) to the right of A.
Distance covered in fixing flag to A_1 = 2 + 2 = 4 m
Distance covered in fixing flag to A_2 = 4 + 4 = 8 m
Distance covered in fixing flag to A_3 = 6 + 6 = 12 m
Distance covered in fixing flag to A_{13} = 26 + 26 = 52 m
This forms an A.P. with,
First term, a = 4
Common difference, d = 4
and n = 13

iii. $\therefore$ Distance covered in fixing 13 flags to the left of A = S_{13}
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$\Rightarrow S_{13} = \frac{13}{2}[2 \times 4 + 12 \times 4]$
= $\frac{13}{2} \times$ [8 + 48]
= $\frac{13}{2} \times$ 56
= 364
Similarly, distance covered in fixing 13 flags to the right of A = 364
Total distance covered by Ruchi in completing the task
= 364 + 364 = 728 m
OR
Maximum distance travelled by Ruchi in carrying a flag
= Distance from A_{13} to A or B_{13} to A = 26 m

37. i. Q(x, y) is mid-point of B(-2, 4) and C(6, 4)
$\therefore$ (x, y) = $\left(\frac{-2+6}{2}, \frac{4+4}{2}\right) = \left(\frac{4}{2}, \frac{8}{2}\right)$ = (2, 4)

ii. Since PQRS is a rhombus, therefore, PQ = QR = RS = PS.
$\therefore$ PQ = $\sqrt{(-2-2)^2 + (1-4)^2}$ = $\sqrt{16+9} = \sqrt{25}$ = 5 units
Thus, length of each side of PQRS is 5 units.

iii. Length of route PQRS = 4 PQ
= 4 $\times$ 5 = 20 units
OR

Length of CD = 4 + 2 = 6 units and length of AD = 6 + 2 = 8 units

$\therefore$ Length of route ABCD - 2(6 + 8) = 28 units

38. i. In $\triangle$ADC,

$\sin 30^\circ = \frac{AD}{AC}$

$\frac{1}{2} = \frac{50}{AC}$

AC = 100 m

Now, In $\triangle$BEC

$\sin 60^\circ = \frac{BE}{BC}$

$\frac{\sqrt{3}}{2} = \frac{60}{BC}$

$BC = \frac{120}{\sqrt{3}}$

$BC = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3}$ m

BC = 40 $\times$ 1.73 = 69.2 m

ii. Clearly DCE is a straight line

$\angle$DCA + $\angle$ACB + $\angle$BCE = 180°

30° + $\angle$ACB + 60° = 180°

$\angle$ACB = 90°

ΔACB is a right triangle.

Now applying Pythagoras theorem, in $\triangle$ABC,

$(AB)^2 = (AC)^2 + (BC)^2$

$(d)^2 = (100)^2 + (40\sqrt{3})^2$

$d^2 = 10000 + 4800$

$d^2 = 14800$

d = $20\sqrt{37}$ m

d = 121.66m