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2025 XII 30 1100 – N 926 – MATHEMATICS (71) GEOMETRY- PART II (E)

(NEW COURSE)

Time : 2 Hours

(Pages 6)

Max. Marks : 40

----MODEL ANSWER----

Q.1 (A) Choose the correct alternative:

4

(1) Ans. (b)

(2) Ans. (d) triangle

(3) Ans. (d)

$$DE \parallel BC$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{x-1} = \frac{x-3}{x-5}$$

$$x(x-5) = (x-3)(x-1)$$

$$x^2 - 5x = x^2 - x - 3x + 3$$

$$-5x = -4x + 3$$

$$-5x + 4x = 3$$

$$-x = 3$$

$$x = -3$$

(4) Ans. (A)

$$\frac{\text{Circumference}}{\text{Area}} = \frac{2}{7}$$

$$\frac{2\pi r}{\pi r^2} = \frac{2}{7}$$

$$\frac{2}{r} = \frac{2}{7}$$

$$r = 7\text{cm}$$

$$\text{Circumference} = 2\pi r = 2 * \pi * 7 = 14\pi\text{cm}$$

(B) Solve the following:

4

(1) Ans. ΔPQS & ΔQRS have common base.

$$\therefore \frac{A(\Delta PQS)}{A(\Delta QRS)} = \frac{PM}{RN} \quad [\Delta s \text{ with equal base}]$$

$$\therefore \frac{100}{110} = \frac{10}{RN}$$

$$\therefore RN = \frac{10 \times 110}{100}$$

$$\therefore RN = 11 \text{ cm.}$$

(2) Ans. If two triangles are similar, then the ratio of the area of both triangles is proportional to the square of the ratio of their corresponding sides.

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$$

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{3^2}{5^2}$$

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{9}{25}$$

(3) Ans. The centroid of a triangle whose vertices are

$$\text{Diameter of a wheel} = 35 \text{ cm} = (x_1, y_1) (x_2, y_2) \text{ and } (x_3, y_3) \text{ is } \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Given

The vertices of a triangle are (4, -3) (7, 5) and (-2, 1)

To find

The y co-ordinate of the centroid of a triangle.

Solutions

The formula for calculating the centroid of the triangle is

$$\text{Centroid} = [(x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3]$$

Now, substituting the values in the above mentioned formula

$$\text{Centroid} = [(4+7-2)/3, (-3+5+1)/3]$$

$$\text{Centroid} = [9/3, 3/3]$$

$$\text{Centroid} = [3, 1]$$

Here, x co-ordinate is 3 and y co-ordinate is 1.

Hence, the y co-ordinate of the centroid of the triangle is 1

(4) Ans. $\text{LHS} = \tan^4 \theta + \tan^2 \theta$

$$= \tan^2 \theta (1 + \tan^2 \theta)$$

$$= (\sec^2 \theta - 1) (\sec^2 \theta)$$

$$\left[\begin{aligned} \because 1 + \tan^2 A &= \sec^2 A \\ \therefore \tan^2 A &= \sec^2 A - 1 \end{aligned} \right]$$

$$\text{LHS} = \sec^4 \theta - \sec^2 \theta$$

$$\text{RHS} = \sec^4 \theta - \sec^2 \theta$$

$$\text{LHS} = \text{RHS.}$$

$$\therefore \tan^4 \theta + \tan^2 \theta = \sec^4 \theta - \sec^2 \theta.$$

Q.2(A) Complete the following activities:(Any TWO)

4

(1) Ans. L.H.S = $\cot \theta + \sin \theta$

$$\begin{aligned} & \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \\ &= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cdot \cos \theta} \\ &= \frac{1}{\sin \theta \cdot \cos \theta} \dots\dots\dots \left[\cos^2 \theta + \sin^2 \theta = 1 \right] \\ &= \frac{1}{\sin \theta} \times \frac{1}{\cos \theta} \\ &= \operatorname{cosec} \theta \times \sec \theta \\ &= \text{R.H.S} \end{aligned}$$

(2) Ans. Let BE = x units [supposition]

$$BC = BE + CE \quad [B - E - C]$$

$$\therefore 6.4 = x + CE$$

$$\therefore CE = (6.4 - x) \text{ units}$$

In $\triangle ABC$, seg DE $\parallel$ side AB [Given]

$$\therefore \frac{AD}{DC} = \frac{BE}{EC}$$

$$\therefore \frac{5}{3} = \frac{x}{6.4 - x}$$

$$\therefore 5(6.4 - x) = 3x$$

$$\therefore 6.4 \times 5 - 5x = 3x$$

$$\therefore 6.4 \times 5 = 3x + 5x$$

$$\therefore \frac{6.4 \times 5}{8} = x$$

$$\therefore x = 4$$

$$\therefore BE = 4 \text{ units}$$

(3) Ans. Given: $\angle ACB = 90^\circ$

$$\angle CDA = 90^\circ$$

CE is angle bisector of $\angle ACB$

$$\text{To Prove} = \frac{AD}{BD} = \frac{AE^2}{BE^2}$$

Proof:

$$\angle ACB = 90^\circ$$

$$\angle CDA = 90^\circ$$

$$\triangle ACB \sim \triangle CDA$$

$$\frac{AC}{CD} = \frac{AB}{AC} = \frac{BC}{AD} \quad \dots\dots\dots \text{(I)}$$

$$\triangle ACB \sim \triangle ADB$$

$$\frac{AC}{CD} = \frac{AB}{CB} = \frac{BC}{BD} \quad \dots\dots\dots \text{(II)}$$

From (I)

$$AC^2 = AB \cdot CD$$

$$\frac{AC^2}{BC^2} = \frac{CD}{BD}$$

$$\dots\dots\dots \text{(III)}$$

CE is the angle bisector

$$\triangle ACE \sim \triangle BCE$$

$$\frac{AC}{BC} = \frac{AE}{BE} \quad \dots\dots\dots \text{(IV)}$$

Now consider $\triangle CDA$

$$CD^2 + AD^2 = AC^2 \quad \dots\dots\dots \text{(2)}$$

In $\triangle ADC$ & $\triangle CDB$

$$\frac{AD}{CD} = \frac{AC}{BC} = \frac{DC}{DB}$$

$$CD^2 = AD \times DB \quad \dots\dots\dots \text{(1)}$$

Put (1) in (2)

$$AD \cdot DB + AD^2 = AC^2$$

$$AC^2 = AD (DB + AD)$$

$$AC^2 = AD (AB) \quad \dots\dots\dots \text{(IV)}$$

Put (V) in (IV)

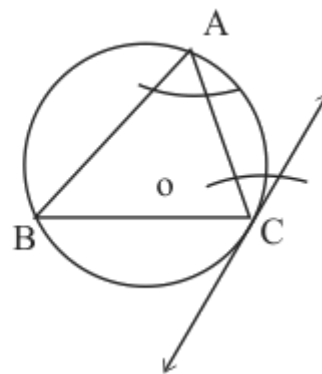
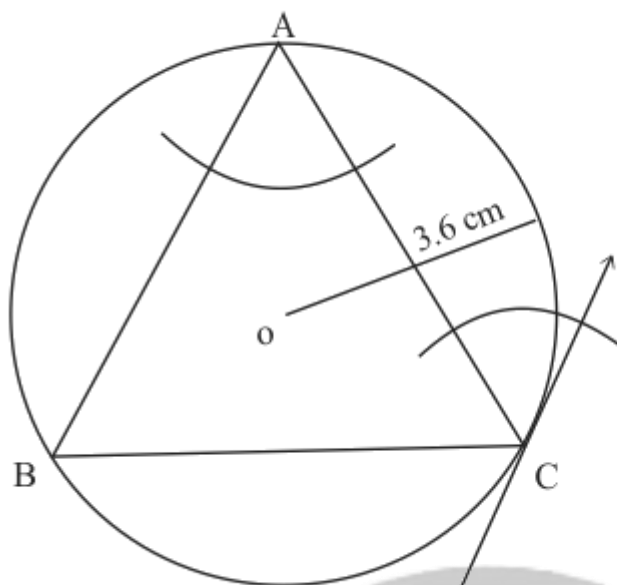
$$\frac{AC^2}{BC^2} = \frac{AE^2}{BE^2}$$

$$\frac{AD \times AB}{AB \times BD} = \frac{AE^2}{BE^2} = \frac{AD}{BD}$$

Hence Proved.

(B) Solve the following: (Any FOUR)

(1) Ans.



(2) Ans. By tangent secant theorem,

$$PQ^2 = PS \times PR$$

$$12^2 = PS \times 8$$

$$PS = \frac{144}{8}$$

$$PS = 18$$

(3) Ans. Seg AB $\parallel$ seg PQ $\parallel$ seg DC [Given]

$$\therefore \frac{AP}{PD} = \frac{BQ}{QC}$$

[Property of intercepts made by three parallel lines.]

$$\therefore \frac{15}{12} = \frac{BQ}{14}$$

$$\therefore \frac{15 \times 14}{12} = BQ$$

$$\therefore BQ = 17.5 \text{ units}$$

(4) Ans. $r = 21 \text{ cm}$, $\theta = 150$, $\pi = \frac{22}{7}$

$$\begin{aligned} \text{Area of the sector, } A &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{150}{360} \times \frac{22}{7} \times 21 \times 21 \\ &= \frac{1155}{2} = 577.5 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Length of the arc, } l &= \frac{\theta}{360} \times 2\pi r \\ &= \frac{150}{360} \times 2 \times \frac{22}{7} \times 21 \\ &= 55 \text{ cm} \end{aligned}$$

(5) Ans. In $\triangle DEF$,
 line $PQ \parallel$ side EF
 $\therefore \frac{DP}{PE} = \frac{DQ}{QF}$ ($\because$ Basic proportionality theorem)
 $\therefore \frac{2.4}{7.2} = \frac{1}{QF}$
 $\therefore QF \times 2.4 = 7.2$
 $\therefore QF = \frac{7.2}{2.4}$
 $\therefore QF = 3$

Q.3(A) Complete the following activity:(Any ONE)

3

(1) Ans. In $\triangle PQR$, $\angle PQR = 90^\circ$, seg $QS \perp$ seg PR
 $QS = \sqrt{PS \times SR}$ (theorem of geometric mean)
 $= \sqrt{10 \times 8}$
 $= \sqrt{5 \times 2 \times 8}$
 $= \sqrt{5 \times 16}$
 $= 4\sqrt{5}$

In $\triangle QSR$, by Pythagoras theorem

$$\begin{aligned} QR^2 &= QS^2 + SR^2 \\ &= (4\sqrt{5})^2 + 8^2 \\ &= 16 \times 5 + 64 \\ &= 80 + 64 \\ &= 144 \end{aligned}$$

$$\therefore QR = 12$$

In $\triangle PSQ$, by Pythagoras theorem

$$\begin{aligned} PQ^2 &= QS^2 + PS^2 \\ &= (4\sqrt{5})^2 + 10^2 \\ &= 16 \times 5 + 100 \\ &= 80 + 100 \\ &= 180 \\ &= 36 \times 5 \end{aligned}$$

$$\therefore PQ = 6\sqrt{5}$$

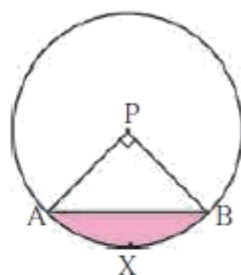
(2) Ans. $m(\text{arc } PR) = m \angle POR$
 [Definition of measure of minor arc]
 $\therefore m(\text{arc } PR) = 70^\circ$ 1
 Chord $PQ \cong$ chord RS [Given]
 $\therefore \text{arc } PQ \cong \text{arc } RS$
 [In a circle, congruent chords have corresponding minor arcs congruent]
 $\therefore m(\text{arc } PQ) = 80^\circ$ 2
 $m(\text{arc } PR) + m(\text{arc } RS) + m(\text{arc } PQ) + m(\text{arc } QS) = 360^\circ$ [Measure of a circle]
 $\therefore 70^\circ + 80^\circ + 80^\circ + m(\text{arc } QS) = 360^\circ$
 $\therefore m(\text{arc } QS) = 360^\circ - 230^\circ$
 $\therefore m(\text{arc } QS) = 130^\circ$ 3
 $m(\text{arc } QSR) = m(\text{arc } QS) + m(\text{arc } SR)$
 [Arc addition property]
 $\therefore m(\text{arc } QSR) = 130^\circ + 80^\circ$
 $\therefore m(\text{arc } QSR) = 210^\circ$

(B) Solve the following: (Any TWO)**(1) Ans.**

$$r = 10 \text{ cm}, \theta = 90^\circ, \pi = 3.14$$

$$\begin{aligned} A(P\text{-}AXB) &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{90}{360} \times 3.14 \times 10^2 \\ &= \frac{1}{4} \times 314 \\ &= 78.5 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A(\triangle APB) &= \frac{1}{2} \text{ base} \times \text{height} \\ &= \frac{1}{2} \times 10 \times 10 \\ &= 50 \text{ cm}^2 \end{aligned}$$



$$\begin{aligned} A(\text{minor segment}) &= A(P\text{-}AXB) - A(\triangle PAB) \\ &= 78.5 - 50 \\ &= 28.5 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A(\text{major segment}) &= A(\text{circle}) - A(\text{minor segment}) \\ &= 3.14 \times 10^2 - 28.5 \\ &= 314 - 28.5 \\ &= 285.5 \text{ cm}^2 \end{aligned}$$

(2) Ans.

$$A(8, 9) = (x_1, y_1)$$

$$B(1, 2) = (x_2, y_2)$$

$$P(k, 7) = (x, y)$$

Let point P divide seg AB in the ratio m : n

By section formula,

$$y = \frac{my_2 + ny_1}{m + n}$$

$$7 = \frac{m \times 2 + n \times 9}{m + n}$$

$$\therefore 7(m + n) = 2m + 9n$$

$$\therefore 7m + 7n = 2m + 9n$$

$$\therefore 5m = 2n$$

$$\therefore \frac{m}{n} = \frac{2}{5}$$

$$\therefore m : n = 2 : 5$$

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$\therefore k = \frac{2 \times 1 + 5 \times 8}{2 + 5}$$

$$\therefore k = \frac{2 + 40}{7}$$

$$\therefore k = \frac{42}{7}$$

$$\therefore k = 6$$

(3) Ans. 1. $MT = 9$ cm [Radius of bigger circle, given]

2. $NT = 2.5$ cm

[Radius of smaller circle, given]

$$MT = MN + NT \quad [M - N - T]$$

$$\therefore 9 = MN + 2.5$$

$$\therefore 9 - 2.5 = MN$$

$$\therefore MN = 6.5 \text{ cm}$$

3. MR is tangent to the smaller circle at point

S [Given]

Seg NS is the radius of the circle

$$\therefore \angle NSM = 90^\circ$$

[Tangent and radius are perpendicular to each other at the point of contact]1

In $\triangle NSM$, $\angle NSM = 90^\circ$ [From 1]

$$\therefore MS^2 + SN^2 = MN^2 \quad [\text{Pythagoras theorem}]$$

$$\therefore MS^2 + 2.5^2 = 6.5^2$$

$$\therefore MS^2 = 6.5^2 - 2.5^2$$

$$\therefore MS^2 = (6.5 + 2.5)(6.5 - 2.5)$$

$$\therefore MS^2 = 9 \times 4$$

$$\therefore MS = 3 \times 2 \quad [\text{Taking square roots}]$$

$$\therefore MS = 6 \text{ cm} \quad \dots 2$$

$$MR = MS + SR \quad [M - S - R]$$

$$\therefore 9 = 6 + SR$$

$$\therefore 9 - 6 = SR$$

$$\therefore SR = 3 \text{ cm} \quad \dots 3$$

$$\frac{MS}{SR} = \frac{6}{3} \quad [\text{From 2 \& 3}]$$

$$\therefore MS : SR = 2 : 1$$

(4) Ans. Region $P - ABC$ is a sector

$$A(P - ABC) = \frac{\theta}{360} \times \pi r^2$$

$$\therefore 154 = \frac{\theta}{360} \times \frac{22}{7} \times 14 \times 14$$

$$\therefore \frac{154 \times 360 \times 7}{22 \times 14 \times 14} = \theta$$

$$\therefore \theta = 90^\circ$$

$$\therefore \angle APC = 90^\circ$$

$$\text{Length of arc } ABC = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{90}{360} \times 2 \times \frac{22}{7} \times 14$$

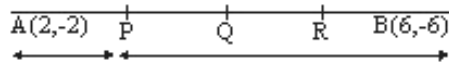
$$\text{Length of arc } ABC = 22 \text{ cm}$$

$$\therefore l(\text{arc } ABC) \text{ is } 22 \text{ cm}$$

Q.4 Solve the following: (Any TWO)**8**

(1) Ans. Let P, Q, R be the three points which divide the line segment joining the points A(2,-2) and B(6,-6) in four equal parts.

Case I - for point P, we have



Hence, $m_1 = 1, m_2 = 3$

$$X_1 = 2, y_1 = -2$$

$$X_2 = 6, y_2 = -6$$

Then coordinates of

$$\begin{aligned} P &= \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \\ &= \frac{1(2) + 3(2)}{1+3}, \frac{1(-6) + 3(-2)}{1+3} \\ &= \frac{2+6}{4}, \frac{-6-6}{4} \\ &= \frac{8}{4}, \frac{-12}{4} = 2, -3 \end{aligned}$$

Case II, for point Q, we have

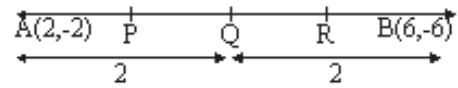
$$x_1 = 2, y_1 = -2$$

$$x_2 = 6, y_2 = -6$$

Then coordinates of

$$\begin{aligned} R &= \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \\ &= \frac{3 \times 6 + 1 \times 2}{3+1}, \frac{3 \times (-6) + 1 \times (-2)}{3+1} \\ &= \frac{18+2}{4}, \frac{-18-2}{4} \\ &= \frac{20}{4}, \frac{-20}{4} \\ &= 5, -5 \end{aligned}$$

Hence the coordinates of the points P, Q & R are (2,-3), (4,-4) and (5, -5) respectively.



Hence, $m_1 = 2, m_2 = 2$

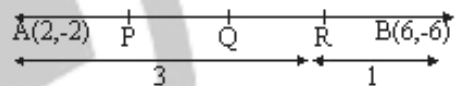
$$x_1 = 2, y_1 = -2$$

$$x_2 = 6, y_2 = -6$$

Then coordinates of

$$\begin{aligned} Q &= \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \\ &= \frac{2(6) + 2(2)}{2+2}, \frac{2(-6) + 2(-2)}{2+2} \\ &= \frac{12+4}{4}, \frac{-12-4}{4} \\ &= \frac{16}{4}, \frac{-16}{4} = 4, -4 \end{aligned}$$

Case III for point R, we have



Hence, $m_1 = 3, m_2 = 1$

(2) Ans. Base area of the cone $= \pi r^2 = 78.5$

$$\therefore r^2 = \frac{78.5}{\pi}$$

$$\therefore r = 5 \text{ cm}$$

$$\begin{aligned}\text{Total surface area of the cone} &= \pi r(l + r) = \pi \times 5(7 + 5) = \pi \times 5 \times 12 \\ &= 188.4\end{aligned}$$

$$\therefore \text{Total surface area of the cone} = 188.4 \text{ cm}^2$$

$$l = \sqrt{h^2 + r^2}$$

$$l^2 = h^2 + r^2$$

$$\begin{aligned}\therefore h^2 &= l^2 - r^2 \\ &= 7^2 - 5^2 \\ &= 49 - 25\end{aligned}$$

$$h^2 = 24$$

$$h = \sqrt{24}$$

$$\therefore h = 2\sqrt{6} \text{ cm}$$

$$\therefore \text{Height of the cone is } 2\sqrt{6} \text{ cm.}$$

(3) Ans. (i) $\angle L = \frac{1}{2} m(\text{arc MN})$ inscribed angle theorem.

$$\therefore 35 = \frac{1}{2} m(\text{arc MN})$$

$$\therefore 2 \times 35 = m(\text{arc MN}) = 70^\circ$$

$$\begin{aligned}\text{(ii) } m(\text{arc MLN}) &= 360^\circ - m(\text{arc MN}) \text{ definition of measure of arc} \\ &= 360^\circ - 70^\circ = 290^\circ\end{aligned}$$

Now, chord LM $\cong$ chord LN

$$\therefore \text{arc LM} \cong \text{arc LN}$$

but $m(\text{arc LM}) + m(\text{arc LN}) = m(\text{arc MLN}) = 290^\circ$ arc addition property

$$m(\text{arc LM}) = m(\text{arc LN}) = \frac{290^\circ}{2} = 145^\circ$$

or, (ii) chord LM $\cong$ chord LN

$$\therefore \angle M = \angle N \text{ isosceles triangle theorem.}$$

$$\therefore 2 \angle M = 180^\circ - 35^\circ = 145^\circ$$

$$\therefore \angle M = \frac{145^\circ}{2}$$

Now, $m(\text{arc LN}) = 2 \times \angle M$

$$= 2 \times \frac{145^\circ}{2} = 145^\circ$$

(1) Ans. Let P and Q be the two positions of the plane and let A be the point of observation.
It is given that angles of elevation of the plane in two positions P and Q from the point A are 60° and 30° respectively.
 $\angle PAB = 60^\circ$ and $\angle QAC = 30^\circ$.

$PB = QC = 3600\sqrt{3}\text{m}$, time taken = 30 sec.

$$\text{In } \triangle PAB, \tan 60^\circ = \frac{BP}{AB}$$

$$\therefore \sqrt{3} = \frac{3600\sqrt{3}}{AB} \quad (\tan 60 = \sqrt{3})$$

$$\therefore AB = 3600 \text{ meters}$$

$$\text{In } \triangle QAC, \tan 30^\circ = \frac{QC}{AC}$$

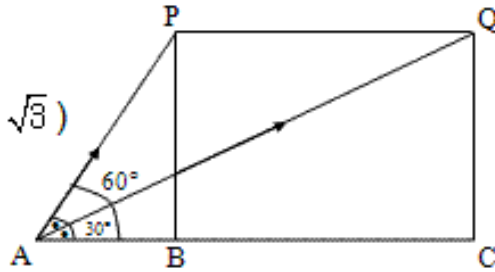
$$\therefore \frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{AC} \quad \left(\tan 30 = \frac{1}{\sqrt{3}} \right)$$

$$\therefore AC = 3600 \times 3 = 10800 \text{ m}$$

$$\therefore PQ = BC = AC - AB = 10800 - 3600 = 7200 \text{ m}$$

Thus the plane travels 7200 m in 30 sec.

$$\begin{aligned} \therefore \text{Speed of the plane} &= \frac{\text{Distance}}{\text{Time}} = \frac{7200}{30} = 240 \text{ m/sec} \\ &= \frac{240}{1000} \times 60 \times 60 = 864 \text{ km/sec} \end{aligned}$$



(2) Ans. Since, S and T trisect the line QR

$$QS = ST = TR$$

$$QS = ST = TR = x$$

$$QT = x + x = 2x$$

$$QR = 2x + x = 3x$$

Now, In $\triangle PQS$

$$PS^2 = PQ^2 + QS^2 \text{ [Pythagoras theorem]}$$

$$PS^2 = PQ^2 + x^2$$

$$\text{In } \triangle PQT, PT^2 = PQ^2 + QT^2$$

$$PT^2 = PQ^2 + (2x)^2$$

$$PT^2 = PQ^2 + 4x^2$$

$$\text{In } \triangle PQR, PR^2 = PQ^2 + QR^2$$

$$PR^2 = PQ^2 + (3x)^2$$

$$PR^2 = PQ^2 + 9x^2$$

$$3PR^2 + 5PS^2 = 3(PQ^2 + 9x^2) + 5(PQ^2 + x^2)$$

$$3PR^2 + 5PS^2 = 3PQ^2 + 27x^2 + 5PQ^2 + 5x^2$$

$$3PR^2 + 5PS^2 = 8PQ^2 + 32x^2$$

$$3PR^2 + 5PS^2 = 8(PQ^2 + 4x^2)$$

$$3PR^2 + 5PS^2 = 8PT^2$$

Hence proved

....All The Best....



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