Solution
PRELIMINARY EXAM -4
Class 10 - Mathematics

SECTION A

1. **(a)** Q
Explanation:
Q

2. **(c)** 60°
Explanation:
60°

3. **(c)** 3
Explanation:
3

4. **(a)** 30°
Explanation:
30°

5. **(c)** $2:1$
Explanation:
$2:1$

6. **(c)** 13 and 12
Explanation:
13 and 12

7. **(b)** 52 is the mode of the data.
Explanation:
52 is the mode of the data.

8. **(c)** $\left(\frac{b}{2}, -a\right)$ as its solution
Explanation:
$\left(\frac{b}{2}, -a\right)$ as its solution

9. **(d)** $\frac{4}{\sqrt{15}}$
Explanation:
$\frac{4}{\sqrt{15}}$

10. **(c)** irrational number
Explanation:
irrational number

11.

(d) 0
Explanation:
0

12.

(b) $x^2 + 1 = 0$
Explanation:
$x^2 + 1 = 0$

13.

(b) 2
Explanation:
2

14.

(b) $3a$
Explanation:
3a

15.

(d) 3 cm
Explanation:
3 cm

16.

(c) 150°
Explanation:
150°

17.

(c) $\frac{3}{4}$
Explanation:
$\frac{3}{4}$

18.

(c) 6 cm
Explanation:
6 cm

19.

(d) A is false but R is true.
Explanation:
A is false but R is true.

20.

(b) Both A and R are true but R is not the correct explanation of A.
Explanation:
Both A and R are true but R is not the correct explanation of A.

SECTION B

21. $m^2 = a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta$

$n^2 = b^2\sec^2\theta + a^2\tan^2\theta + 2ab\sec\theta\tan\theta$

$m^2 - n^2 = a^2\left(\sec^2\theta - \tan^2\theta\right) + \ b^2\left(\tan^2\theta - \sec^2\theta\right)$

$\Rightarrow m^2 - n^2 = a^2 - b^2$ or $a^2 + n^2 = m^2 + b^2$

OR

$\sin^2\ A + \cos^2\ A = 1$

Dividing both sides by $\cos^2\ A$, we get

$\frac{\sin^2\ A}{\cos^2\ A} + \frac{\cos^2\ A}{\cos^2\ A} = \frac{1}{\cos^2\ A}$

$\tan^2\ A + 1 = \sec^2\ A$

$\tan^2\ A + 1 = \left(\frac{5}{3}\right)^2$

$\tan A = \frac{4}{3}$

22. Let $P(x, y)$ be equidistant from $A(7,1)$ and $B(3,5)$

$PA = PB \Rightarrow PA^2 = PB^2$

$(x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$

$x^2 + 49 - 14x + y^2 + 1 - 2y \ = x^2 + 9 - 6x + y^2 + 25 - 10y$

$x = 2 + y$

Thus, abscissa of the point P is 2 more than its ordinate.

23. $\frac{AP}{AB} = \frac{1}{3}; \frac{AQ}{AC} = \frac{1.5}{4.5} = \frac{1}{3}$

$\angle ACP = \angle ACB$

$\triangle APQ \sim \triangle ABC$

$PQ = 1.2\ cm$

24. Total possible outcomes $= 90$

i. Number of favourable outcomes for a 2-digit number $= 82$

$P($2-digit number$) = \frac{82}{90}$ or $\frac{41}{45}$

ii. Number of favourable outcomes for multiple of $1 = 90$

$P($ a number multiple of $1) = \frac{90}{90}$ or 1

25. Adding equations we get

$x + y = 3$

Subtracting equations we get

$-x + y = -1$

Solving to get

$x = 2$ and $y = 1$

OR

Let smaller angle be x and greater angle be y

ATQ, $x + y = 180$

Also $y = x + 50$

Solving we get

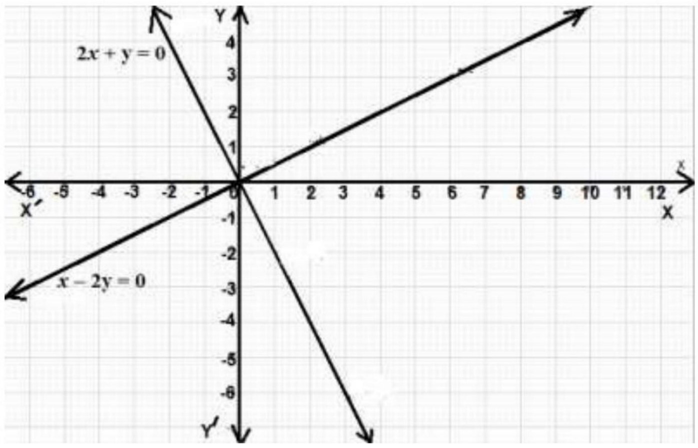
$x = 65^\circ$ and $y = 115^\circ$

SECTION C

26. $\frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{-2}{1} = -2$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ System of equation is consistent.



Solution is $(0,0)$ or $x=0$ and $y=0$

27. Diagonals of a parallelogram bisect each other.

$\therefore$ Co-ordinates of mid point of diagonal $AC = Co$-ordinates of mid-point of diagonal BD.

$\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{p+7}{2}, \frac{2+q}{2}\right)$

$\Rightarrow \frac{p+7}{2} = \frac{15}{2}$ and $\frac{2+q}{2} = \frac{5}{2}$

$\therefore p = 8$ and $q = 3$

Diagonal $AC = \sqrt{3^2+3^2} = 3\sqrt{2}$

Diagonal $BD = \sqrt{(-1)^2+1^2} = \sqrt{2}$

$AC \neq BD \therefore ABCD$ is not a rectangle.

28. LHS $= \frac{\cos\theta - 2\cos^3\theta}{\sin\theta - 2\sin^3\theta} + \cot\theta$

$= \frac{\cos\theta(1-2\cos^2\theta)}{\sin\theta(1-2\sin^2\theta)} + \cot\theta$

$= \frac{\cos\theta}{\sin\theta}\left[\frac{\sin^2\theta+\cos^2\theta-2\cos^2\theta}{\sin^2\theta+\cos^2\theta-2\sin^2\theta}\right] + \cot\theta$

$= \frac{\cot\theta(\sin^2\theta-\cos^2\theta)}{(\cos^2\theta-\sin^2\theta)} + \cot\theta$

$= -\cot\theta + \cot\theta$

$= 0 =$ RHS

OR

Given: $\sin\theta + \cos\theta = x$

Squaring both sides

$\sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta = x^2$

$2\sin\theta\cos\theta = x^2 - 1$

RHS $= \frac{2-(2\sin\theta\cos\theta)^2}{2}$

$= \frac{2-4\sin^2\theta\cos^2\theta}{2}$

$= 1 - 2\sin^2\theta\cos^2\theta$

$= \left(\sin^2\theta+\cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta$

$= \left(\sin^4\theta+\cos^4\theta\right) =$ LHS

29. $\angle QPT = 75^\circ$

$\angle PQT = 75^\circ$

$\theta = 30^\circ$

$\sin 2\theta = \sin 2(30^\circ)$

$= \sin 60^\circ = \frac{\sqrt{3}}{2}$

30. Let $\sqrt{5}$ is a rational number.

$\sqrt{5} = \frac{a}{b}$, (a, b are co-primes and b$\neq$0)

or, $a = b\sqrt{5}$

On squaring both the sides, we get

$a^2 = 5b^2$ ....(1)
Hence 5 is a factor of a^2
so 5 is a factor of a
Let a = 5c, (c is some integer)
$\therefore a^2 = 25c^2$
From equation(1) putting the value of a^2 gives
$5b^2 = 25c^2$
or $b^2=5c^2$
so 5 is a factor of b^2
or 5 is a factor of b
Hence 5 is a common factor of a and b
But this contradicts the fact that a and b are co-primes.
This is because we assumed that $\sqrt{5}$ is rational
$\therefore \sqrt{5}$ is irrational.

OR

$p.q.r + q = q(pr + 1)$
Thus, the given number has more than 2 factors.
Hence it is composite.

i. Taking $p = 3, q = 5$ and $r = 7$
   $pqr + 1 = 3.5.7 + 1 = 106$ is a composite number
ii. Taking $p = 2, q = 3$ and $r = 5$
   $pqr + 1 = 2.3.5 + 1 = 31$ is a prime number

31. $p(x) = 4x^2 + 3x - 1$
Zeroes are $\frac{1}{4}, -1$
New zeroes 4, -1
Sum of new zeroes $= 4 + (-1) = 3$
Product of zeroes $= 4 \times (-1) = -4$
Required polynomial is $\left(x^2 - 3x - 4\right)$

SECTION D

32. **Converse of the Basic Proportionality Theorem (Thales)**
Statement (converse).
Let $\triangle ABC$. If a line through points D on AB and E on AC (with D and E distinct from A, B, C) satisfies
$\frac{AD}{DB} = \frac{AE}{EC}$,
then the line DE is parallel to BC.
Proof:
Consider $\triangle ABC$ with $D \in AB$ and $E \in AC$ such that $\frac{AD}{DB} = \frac{AE}{EC}$.
Through D draw a line parallel to BC; let it meet AC at F. By the (direct) Basic Proportionality Theorem, since $DF \| BC$ we have
$\frac{AD}{DB} = \frac{AF}{FC}$.
But we are given $\frac{AD}{DB} = \frac{AE}{EC}$.
Thus, $\frac{AF}{FC} = \frac{AE}{EC}$.
On the line AC the only way two ratios of the form $\frac{\text{segment from } A}{\text{remaining segment}}$ can be equal is if the dividing points coincide; therefore F and E are the same point. Hence the line we drew through D parallel to BC actually passes through E. That is, $DE \| BC$.

OR

$\triangle ABC \sim \triangle DAC$ ...(1)
Similarly, $\triangle ABC \sim \triangle DBA$ ...(2)

From equations (1) and (2)

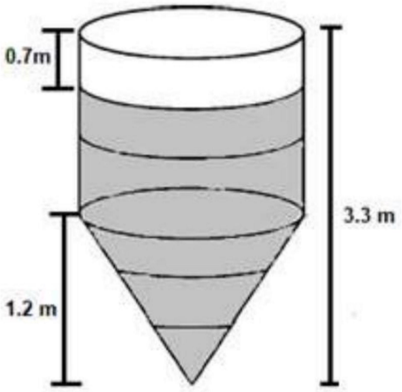
$\Delta DAC \sim \Delta DBA$ or $\Delta ADB \sim \Delta CDA$

$\frac{AD}{CD} = \frac{BD}{AD}$

$AD^2 = BD \times CD$

$= 8 \times 2$

$\therefore AD = 4\ cm$

33. 

Diameter $= 1\ m$

$r = 0.5\ m$

Height of Cylinder $(H) = 3.3 - 1.2 = 2.1\ m$

Capacity of the tank = Volume of cylinder + Volume of cone

$= \frac{22}{7} \times (0.5)^2 \times 2.1 + \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times 1.2$

$= 1.96 m^3$

Slant height $(l) = \sqrt{(1.2)^2 + (0.5)^2} = 1.3\ m$

Height of cylindrical part in contact with liquid $= 2.1 - 0.7 = 1.4\ m$

Surface area of tank in contact with liquid

= Curved Surface Area of Cylindrical part in contact with liquid + Curved surface Area of cone

$= 2 \times \frac{22}{7} \times 0.5 \times 1.4 + \frac{22}{7} \times 0.5 \times 1.3$

$= 6.44 m^2$ (approx .)

34.

Number of lions	**Number of regions**	**Cumulative frequency**
0-100	2	2
100-200	5	7
200-300	9	16
300-400	12	28
400-500	x	$28 + x$
500-600	20	$48 + x$
600-700	15	$63 + x$
700-800	9	$72 + x$
800-900	y	$72 + x + y$
900-1000	2	$74 + x + y$
	100	

$74 + x + y = 100$

$x + y = 26$

Median class is $500 - 600$

$525 = 500 + \left[\frac{\frac{50}{2} - (28+x)}{20}\right] \times 100$

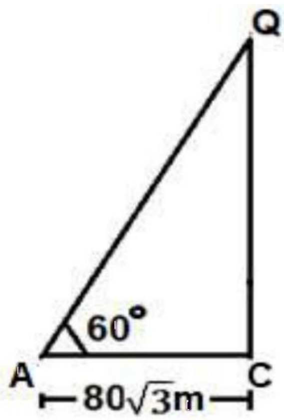
On solving, we get $x = 17$
$y = 9$

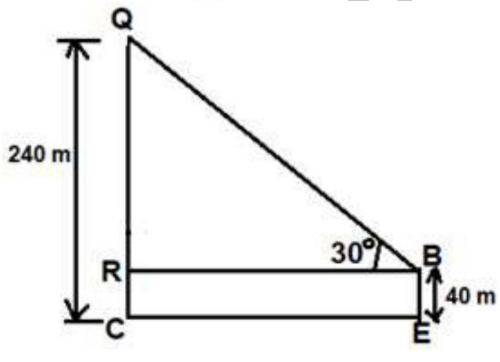
35. Let distance of gate at P from point B is $x\ m$
Then distance of gate at P from point A is $(35 + x)m$
In right $\triangle APB$
$(x + 35)^2 + x^2 = (65)^2$
$x^2 + 35x - 1500 = 0$
$(x + 60)(x - 25) = 0$
$x = 25$
Hence, $x + 35 = 60$
Distance of P from $A = 60\ m$
Distance of P from $B = 25\ m$

OR

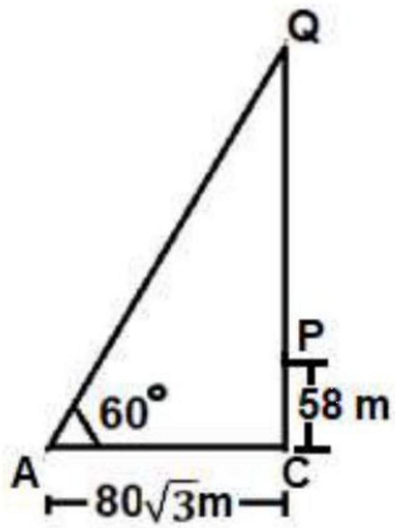
For real roots, $D \geq 0$
$[-2(p+1)]^2 - 4p^2 \geq 0$
$\Rightarrow p \geq -\frac{1}{2}$
$\therefore$ smallest value of $p = -\frac{1}{2}$
At $p = -\frac{1}{2}$ given equation becomes
$x^2 - 2\left(\frac{-1}{2} + 1\right)x + \left(\frac{-1}{2}\right)^2 = 0$
$x^2 - x + \frac{1}{4} = 0$ or $4x^2 - 4x + 1 = 0$
$(2x - 1)(2x - 1) = 0$
$\therefore$ roots are $\frac{1}{2}, \frac{1}{2}$

SECTION E

36. i.


ii.


iii. a.



In $\triangle ACQ$

$\frac{QC}{AC} = \tan 60^\circ = \sqrt{3}$

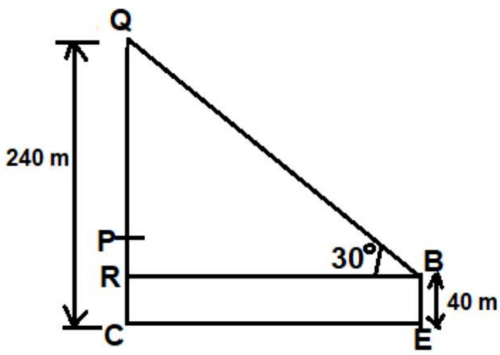
$QC = 240\ m$

Height of statue including base $= 240\ m$

Height of statue excluding base $= 240 - 58 = 182\ m$

OR

b.

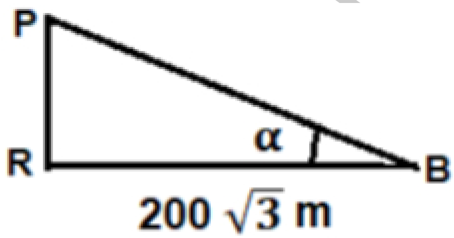


$QR = 240 - 40 = 200\ m$

In $\triangle$QRB

$\frac{QR}{RB} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

Horizontal distance RB $= 200\sqrt{3}\ m$



In $\triangle PRB$

$\tan\alpha = \frac{PR}{BR}$

$= \frac{18}{200\sqrt{3}}$ or $\frac{3\sqrt{3}}{100}$

37. i. $\angle POA = 120^\circ$

ii. Length of wire needed to fence entire piece of land $= \frac{22}{7} \times 35 + 70 = 180\ m$

iii. a. Required area $= \frac{60}{360} \times \frac{22}{7} \times (35)^2 - \frac{\sqrt{3}}{4} \times (35)^2$

$= \left(\frac{1925}{3} - \frac{1225\sqrt{3}}{4}\right) m^2$ or $111.89\ m^2$ (approx.)

OR

b. In $\triangle APB, \frac{AP}{AB} = \cos 30^\circ$

$AP = 35\sqrt{3}\ m$

Required length of wire $= \frac{120}{360} \times 2 \times \frac{22}{7} \times 35 + 35\sqrt{3}$

$= \left(\frac{220}{3} + 35\sqrt{3}\right) m$ or $133.8\ m$ (approx.)

38. Here AP is $400, 407.6, 415.2, \ldots$

i. $a_6 = 400 + 5(7.6) = 438\ m$

ii. $a_8 - a_4 = 30.4\ m$

iii. a. $S_6 = \frac{6}{2}(2 \times 400 + 5 \times 7.6)$

$= 2514\ m$

OR

b. Total distance covered $= S_8 - S_3$

$= \frac{8}{2}(2 \times 400 + 7 \times 7.6) \ -\frac{3}{2}(2 \times 400 + 2 \times 7.6)$

$= 2190\ m$