Solution
PRELIMINARY EXAM - 5
Class 10 - Mathematics
Section A

1. **(a)** 1680
Explanation:
LCM = Product of greatest power of each prime factor involved in the numbers
= $2^4 \times 3 \times 5 \times 7$
= $16 \times 3 \times 5 \times 7$
= 1680

2. **(b)** 4
Explanation:
Number of zeros= number of times the graph touches x-axis.
Here the graph touches x-axis 4 times.

3. **(a)** no solution
Explanation:
Since, we have y = 0 and y = -6 are two parallel lines.
therefore, no solution exists.

4. **(d)** 0.1
Explanation:
x^2 - 0.9k = 0
$(0.3)^2$ - 0.9k = 0
0.09 - 0.9k = 0
Now, let's isolate k
-0.9k = -0.09
Divide both sides by -0.9 to solve for k
k = $\frac{(-0.09)}{(-0.9)}$
k = 0.1

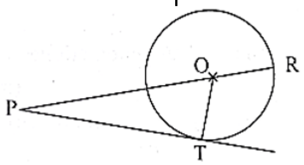
5. **(b)** 3
Explanation:
Given
k + 2, 4k - 6 and 3k - 2 are three consecutive terms of an A.P.
Then (4k - 6) - (k + 2) = (3k - 2) - (4k - 6)
3k - 8 = -k + 4
4k = 8 + 4 = 12
k = $\frac{12}{4}$ = 3

6. **(c)** 5 units
Explanation:
Let be point A(0, 5) and B(-3, 1)
$x_1 = 0, y_1 = 5, x_2 = -3, y_2 = 1$
Distance between the points
AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AB = $\sqrt{(-3-0)^2+(1-5)^2}$
$=\sqrt{9+16}$
$=\sqrt{25}$
= 5 units

7. **(a)** -2
Explanation:
-2

8. **(a)** 8 cm
Explanation:
AD = 5 cm, BC = 12 cm
BD = 2.5 cm
AB = AD + BD
= 5 + 2.5
= 7.5 cm
Now DE || BC
By BPT
$\frac{AD}{AB}=\frac{DE}{BC}$
$\frac{5}{7.5}=\frac{DE}{12}$
$\frac{12\times5}{7.5}=DE$
DE = 8 cm

9.
(c) 32 cm
Explanation:
The correct option is 32 cm.


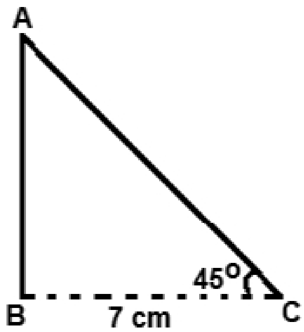
In $\triangle$ POT, $\angle T$ is 90^0,
Therefore, $\triangle$POT is right angle triangle
$(OP)^2 = (OT)^2 + (PT)^2$[using pythagoras theorem]
$OP^2 = (7)^2 + (24)^2$
$OP^2 = (25)^2$
OP = 25 cm
Now, PR = OP + OR
= 25 + 7
= 32 cm

10.
(b) 4 cm
Explanation:
4 cm

11.
(d) a^2b^2
Explanation:
Given: x = a cosθ and y = b sinθ
$\therefore$ $b^2x^2 + a^2y^2$

= $b^2(a \cos\theta)^2 + a^2(b \sin\theta)^2$
= $b^2a^2 \cos^2\theta + a^2b^2\sin^2\theta$
$\Rightarrow b^2x^2 + a^2y^2$
= $a^2b^2(\cos^2\theta + \sin^2\theta)$
= a^2b^2
[$\because \sin^2\theta + \cos^2\theta = 1$]

12. **(a)** $\sqrt{6}$
Explanation:
2 cos45° cot30°
= $2 \times \left(\frac{1}{\sqrt{2}}\right) \times (\sqrt{3})$
= $\frac{2\sqrt{3}}{\sqrt{2}}$
= $\sqrt{6}$

13. **(a)** 7 m
Explanation:


height of tree = AB
$\tan 45^\circ = \frac{AB}{BC}$
$1 = \frac{AB}{7}$
AB = 7 m

14.
(b) 22 cm
Explanation:
Arc length =$\frac{2\pi r\theta}{360} = \left(2 \times \frac{22}{7} \times 21 \times \frac{60}{360}\right) \mathrm{cm} = 22 \mathrm{cm}$

15.
(b) 35
Explanation:
35

16.
(c) $\frac{5}{6}$
Explanation:
$\frac{5}{6}$

17. **(a)** $\frac{12}{13}$
Explanation:
Total events = 52
cards Probability of card which is not in ace Number of card = 52 - 4 = 48
$\therefore$ Probability = $\frac{48}{52} = \frac{12}{13}$

18.

(c) 30

Explanation:

30

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both are correct. Reason is the correct reasoning for Assertion.

Assertion, S_{10} = $\frac{10}{2}[2(-0.5)+(10-1)(-0.5)]$

= 5[-1 - 4.5]

= 5(-5.5) = -27.5

Section B

21. LHS = $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$

= $\frac{(\sec\theta-\tan\theta)+(\sec^2\theta-\tan^2\theta)}{1+\sec\theta+\tan\theta}$

= $\frac{(\sec\theta-\tan\theta)(1+\sec\theta+\tan\theta)}{(1+\sec\theta+\tan\theta)}$

= sec θ - tan θ = $\frac{1-\sin\theta}{\cos\theta}$ = RHS

OR

L.H.S = $\frac{\frac{\cos^2\theta}{\sin^2\theta}\left(\frac{1}{\cos\theta}-1\right)}{1+\sin\theta}$ = $\frac{\cos\theta}{(1+\cos\theta)(1+\sin\theta)}$

R.H.S = $\frac{1}{\cos^2\theta}\times\frac{(1-\sin\theta)\cos\theta}{1+\cos\theta}$ = $\frac{(1-\sin\theta)\cos\theta}{(1-\sin^2\theta)(1+\cos\theta)}$

= $\frac{\cos\theta}{(1+\cos\theta)(1+\sin\theta)}$

∴ L.H.S = R.H.S

22. Let us assume √3 be a rational, then as every rational can be represented in the form p/q where q≠0

Let √3=p/q where p,q have no common factor.

Now squaring on both sides we get $3=p^2/q^2$

$\implies$ $3\times q^2=p^2$

Which means 3 divides p^2 which implies 3 divides p

Hence we can write p=3×k, where k is some constant.

This gives $3\times q^2=9\times k^2$

$q^2=3\times k^2$

Which means 3 divides q^2 which implies 3 divides q.

3 divides p and q which means 3 is a common factor for p and q.

And this is a contradiction for our assumption that p and q have no common factor...

Hence we can say our assumption that √3 is rational is wrong...

And therefore √3 is an irrational...

23. $\angle ABD=\angle ACD \Rightarrow$ AB = AC

$\therefore \frac{BC}{BD}=\frac{BE}{AC}\Rightarrow\frac{BC}{BD}=\frac{BE}{BA}$

$\angle B=\angle B$ (common)

$\therefore \triangle \mathrm{ABD}\sim\quad\triangle \mathrm{EBC}$ [SAS similarity criterion]

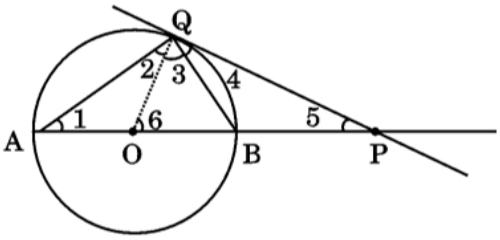
24. Area of circle = 3.14 × 10 × 10 = 314 cm^2

Area of minor sector = $\frac{3.14\times10\times10\times90}{360}=\frac{157}{2}$ cm^2 or 78.5 cm^2

Area of major sector = 314 - 78.5 = 235.5 cm^2

OR

Radius of circle (r) = OA = 7 cm.
Area of the semicircle = $\frac{1}{2} \times \pi r^2$
$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$
= 11 × 7
= 77 cm^2
Area of △ABC = $\frac{1}{2}$ × base × height
= $\frac{1}{2}$ × 14 × 7
= 49 cm^2
∴ Area of the shaded portion = Area of semicircle - Area of the △ABC
= 77 - 49
= 28 cm^2

25. 

Join OQ
OQ = OA
$\Rightarrow \angle 2 = 30^\circ$
$\angle 3 = 90^\circ - 30^\circ = 60^\circ$
$\angle 4 = 90^\circ - 60^\circ = 30^\circ$
$\angle 6 = \angle 1 + \angle 2 = 60^\circ$
Hence $\angle 5 = 90^\circ - 60^\circ = 30^\circ = \angle 4$
∴ BP = BQ

Section C

26. The value of $a = 7 \times 11 \times 13 + 13$.
First calculate $7 \times 11 \times 13 = 1001$.
So $a = 1001 + 13 = 1014$, which is divisible by 2 .
Therefore, a **is a composite number.**
The value of $b = 6 \times 5 \times 4 + 4$.
First calculate $6 \times 5 \times 4 = 120$.
So $b = 120 + 4 = 124$, and this is also divisible by 2 .
Therefore, b **is a composite number.**
The value of $c = 7 \times 13 + 6$.
Calculate $7 \times 13 = 91$.
So $c = 91 + 6 = 97$.
Since 97 is a prime number, **c is not composite.**
Final Answer: a **and** b **are composite;** c **is prime.**
27. Required polynomial is $x^2 + 10x + 24$
For zeroes: $x^2 + 10x + 24 = (x + 6)(x + 4)$
Zeroes are $-6, -4$
28. LHS = (sinA + cosecA)^2 + (cosA + secA)^2
= sin^{2}A + cosec^{2}A + 2 sinA cosecA + cos^{2}A + sec^{2}A + 2 cosA secA
= sin^{2}A + cos^{2}A + cosec^{2}A + sec^{2}A + 2 + 2
= 1 + cosec^{2}A + sec^{2}A + 4
= (1 + cot^{2}A) + (1 + tan^{2}A) + 5
= 7 + tan^{2}A + cot^{2}A = RHS
29. Let the given AP contains n terms.
First term, a=5

Last term, l=45

S_n=400

$\Rightarrow \frac{n}{2}$[a+l]=400

$\Rightarrow \frac{n}{2}$[5+45]=400

$\Rightarrow$n$\times$50=800

$\Rightarrow$n=16

Thus, the given AP contains 16 terms.

Let d be the common difference of the given AP.

then,

T_{16}=45

$\Rightarrow$a+15d=45

$\Rightarrow$5+15d=45

$\Rightarrow$15d=40

$\Rightarrow$d=$\frac{40}{15}=\frac{8}{3}$.

Therefore, common difference of the given AP is $\frac{8}{3}$.

OR

First APs

63, 65, 67, ...

Here, a = 63

d = 65 - 63 = 2

$\therefore$ nth term = 63 + (n - 1)2 $\because$ a_n = a + (n - 1)d

Second APs

3, 10, 17, ...

Here, a = 3

d = 10 - 3 = 7

$\therefore$ nth term = 3 + (n - 1)7 $\because$ a_n = a+(n - 1)d

If the n th terms of two APs are equal then

63 + (n - 1)2 = 3 + (n - 1)7

$\Rightarrow$ (n - 1)2 - (n - 1)7 = 3 - 63

$\Rightarrow$ (n - 1) (2 - 7) = -60

$\Rightarrow$ (n - 1) (-5) = -60

$\Rightarrow n-1=\frac{-60}{-5}$

$\Rightarrow$ n - 1 = 12

$\Rightarrow$ n = 12 + 1

$\Rightarrow$ n = 13

Hence, for n = 13th terms of the two APs are equal

30. By using the step-deviation method:

Class interval	f_i	x_i	$d_i = x_i - 50$	$u_i=\frac{d_i}{20}$	$f_i \times u_i$
0 – 20	17	10	–40	–2	–34
20 – 40	28	30	–20	–1	–28
40 – 60	32	50(=a)	0	0	0
60 – 80	24	70	20	1	24
80 – 100	19	90	40	2	38
Total	120				0

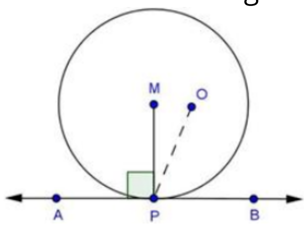
taking a = 50, h = 20

$\bar{x}=a+\frac{\sum f_i u_i}{\sum f_i}\times h$

$\therefore$ Mean = $50 + \frac{0}{120} \times 20$

= 50

31. Let APB be the tangent and take O as centre of the circle.



Let us suppose that MP$\perp$AB does not pass through the centre.

Then,

$\angle OPA = 90^\circ$ [$\because$ Tangent is perpendicular to the radius of circle]

But $\angle MPA = 90^\circ$ [Given]

$\therefore \angle OPA = \angle MPA$

This is only possible when point O and point M coincide with each other.

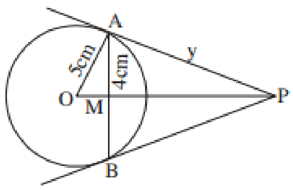
Hence, the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

OR

According to the question, radius of circle is = 5 cm.

Also, AB = 8 cm

Now, AM = $\frac{AB}{2}$ $=$4 cm



In $\triangle$OMA,

By using pythagoras theorem, we get

$OA^2 = OM^2 + AM^2$

$\therefore$ OM = $\sqrt{5^2 - 4^2}$ = 3 cm

Let AP = y cm, PM = x cm

In $\triangle$OAP,

By using pythagoras theorem, we get

$OP^2 = OA^2 = AP^2$

$(x + 3)^2 = y^2 + 25$

$\Rightarrow x^2 + 9 + 6x = y^2 + 25$ ...(i)

In $\triangle$AMP,

By using pythagoras theorem, we get

$x^2 + 4^2 = y^2$ ....(ii)

Substituting eq.(ii) in eq.(i), we get

$\Rightarrow x^2 + 6x + 9 = x^2 + 16 + 25$

$\Rightarrow$ 6x = 32

$\Rightarrow$ x = $\frac{32}{6}$=$\frac{16}{3}$ cm

Now, $y^2 = x^2 + 16 = \frac{256}{9} + 16 = \frac{400}{9}$

$\Rightarrow$ y = $\frac{20}{3}$ cm.

Therefore, length of AP = y = $\frac{20}{3}$ cm.

Section D

32. Let AD be the multistoried building of height hm. And the angle of depression of the top and bottom are 30^o and 45^o.

We assume that BE = 6, CD = 6 and BC = x, ED = x and AC = h - 6.

Here we have to find height and distance of the building.
We use trigonometric ratio.
In △AED,
⇒ tan E = $\frac{AD}{DE}$
⇒ tan45^o = $\frac{AD}{DE}$
⇒ 1 = $\frac{h}{x}$
⇒ x = h
Again in △ABC
⇒ tan B = $\frac{AC}{BC}$
⇒ tan 30^o = $\frac{h-6}{x}$
⇒ $\frac{1}{\sqrt{3}} = \frac{h-6}{x}$
⇒ $h\sqrt{3} - 6\sqrt{3}$ = x
⇒ $h\sqrt{3} - 6\sqrt{3}$ = h
⇒ $h(\sqrt{3} - 1) = 6\sqrt{3}$
⇒ $h = \frac{6\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$
⇒ $h = \frac{18+6\sqrt{3}}{2}$
⇒ h = 9 + $3\sqrt{3}$
3(3 + $\sqrt{3}$)
Hence the required height is 3(3 + $\sqrt{3}$) meter and distance is 3(3 + $\sqrt{3}$) meter

33.

C.I.	f	CF
$100-120$	8	8
$120-140$	9	17
$140-160$	12	29
$160-180$	5	34
$180-200$	6	40
	40	

median class: 140 - 160
Median $= \ell + \frac{\frac{N}{2}-cf}{f} \times h$
$= 140 + \frac{20-17}{12} \times 20$
$= 145$
∴ The median length of the leaves is 145 mm

34. For real and equal roots, $D = 0$
$\therefore [-(p+1)]^2 - 4(p+4) = 0$
$\Rightarrow p^2 - 2p - 15 = 0$
$\Rightarrow (p-5)(p+3) = 0$
$\therefore p = 5, -3$
For $p = 5$,
$9x^2 - 6x + 1 = 0$
$\Rightarrow (3x-1)(3x-1) = 0$
$\therefore x = \frac{1}{3}, \frac{1}{3}$
For $p = -3$,
$x^2 + 2x + 1 = 0$
$\Rightarrow (x+1)(x+1) = 0$
$\therefore x = -1, -1$
Hence roots are $\frac{1}{3}, \frac{1}{3}$ and $-1, -1$ for $p = 5$ and $p = -3$ respectively.

OR

Let breadth be x m and length be $(2x+1)$ m

A.T.Q.

$(2x+1)x=300$
$2x^2+x-300=0$
$(x-12)(2x+25)=0$
$x=12$
$\left(\text{Rejecting } x=\frac{-25}{2}\right)$

length $=25\ m$ and breadth $=12\ m$

35. Radius of a spherical bead $=R=2.1\ mm$

radius of cylinder $=r=1\ mm$

height of cylinder $=h=4.2\ mm$

Volume of wood left in a bead

$=$ Volume of sphere - Volume of cylinder

$=\frac{4}{3}\pi R^3-\pi r^2\ h$

$=\frac{4}{3}\times\frac{22}{7}\times 2.1\times 2.1\times 2.1-\frac{22}{7}\times 1\times 1\times 4.2$

$=25.608 cu.mm$

OR

Edge of cube $=a=3.5\times 2=7\ cm$

Total surface area of solid

$=6a^2+2\pi r^2-\pi r^2$
$=6a^2+\pi r^2$
$=6\times 7\times 7+\frac{22}{7}\times 3.5\times 3.5$
$=\frac{665}{2}$ sq. cm or 332.5 sq. cm

Section E

36. i. Let the fixed charge be ₹ x and per kilometer charge be ₹ y

$\therefore$ x + 10y = 105 ...(i)

x + 15y = 155 ...(ii)

From (i) and (ii)

5y = 50

y = $\frac{50}{5}$ = 10

From equation (i)

x + 100 = 105

x = 105 - 100 = 5

Fixed charges = ₹ 5

ii. Let the fixed charge be ₹ x and per kilometer charge be ₹ y

$\therefore$ x + 10y = 105 ...(1)

x + 15y = 155 ...(2)

From (1) and (2)

5y = 50

y = $\frac{50}{5}$ = 10

From equation (1)

x + 100 = 105

x = 105 - 100 = 5

Per km charges = ₹ 10

iii. Let the fixed charge be ₹ a and per kilometer charge be ₹ b

a + 10b

20 + 10 $\times$ 10 = ₹ 120

OR

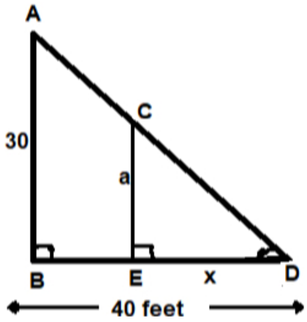
Total amount = x + 10y + x + 25y

= 2x + 35y

= 2 $\times$ 5 + 35 $\times$ 10

= 10 + 350

= ₹ 360

37. i. 

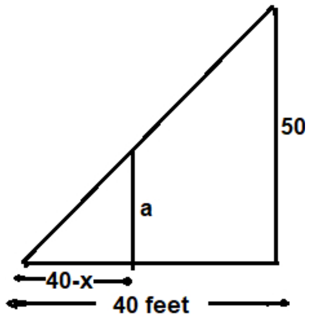
$\triangle$ABD $\sim$ $\triangle$CED (by AA criteria)

$\frac{30}{a} = \frac{40}{x}$

$\frac{x}{a} = \frac{40}{30}$

a = $\frac{30}{40}x$

Again



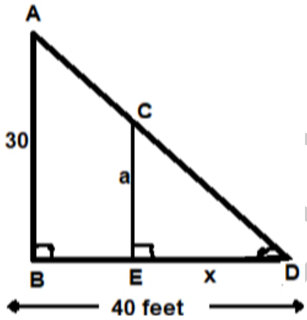
$\frac{40-x}{40} = \frac{a}{50}$

$\frac{40-x}{40} = \frac{30\times x}{40\times 50}$

8000 - 200x = 120x

8000 = 320x

$\therefore$ x = 25 feet

ii. 

$\frac{x}{40} = \frac{a}{30}$

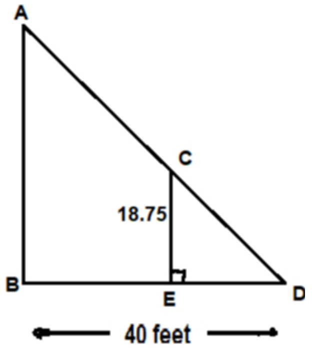
$\frac{25}{40} = \frac{a}{30}$

$\frac{25\times 30}{40} = a$

$\frac{75}{4}$ = a

a = 18.75 feet

iii.



AD = $\sqrt{30^2 + 40^2}$

$= \sqrt{900 + 1600}$

$= \sqrt{2500}$

AD = 50 feet

In $\triangle$CED

CD = $\sqrt{18.75^2 + 25^2}$

$= \sqrt{976.5625}$

= 31.25 feet

AC = AD - CD

= 50 - 31.25

= 18.75 feet

OR

$\sqrt{40^2 + 20^2}$

$= \sqrt{1600 + 400}$

$= \sqrt{2000}$

$= 20\sqrt{5}$ feet

38. i. Coordinates of Q are (9, 5).

$\therefore$ Distance of point Q from y-axis = 9 units

ii. Coordinates of point U are (8, 2).

iii. We have, P(2, 5) and Q (9, 5)

$\therefore$ PQ = $\sqrt{(2-9)^2 + (5-5)^2}$ = $\sqrt{49+0}$ = 7 units

OR

Length of TU = 5 units and of TL = 2 units

$\therefore$ Perimeter of image of a rectangular face = 2(5 + 2) = 14 units